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Improved Surface Layer Wind Modeling for Autonomous Parafoils in a Maritime Environment

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This paper investigates the use of atmospheric boundary layer theory to produce more accurate wind estimates for guiding an autonomous parafoil during the last portion of its flight before touchdown. The problem of wind estimation for a prototype autonomous parafoil aerial delivery system is first explained, followed by the simple assumptions for wind estimation that its guidance algorithm makes. A logarithmic wind profile model in the atmospheric surface layer is then introduced. The parameters and limitations of this model are discussed, along with the characteristics of this model that make it especially useful over the surface of the ocean. Finally, the incorporation of this model into the guidance algorithm of the prototype aerial delivery system is discussed, and subsequently evaluated in flight tests against the original algorithm that did not include the logarithmic surface layer wind model.

Nomenclature

ABL	Atmospheric Boundary Layer
ADS	Aerial Delivery System
AGL	Above Ground Level
CARP	Computed Air Release Point
NPS	Naval Postgraduate School
RLS	Recursive Least Squares
TIP	Turn Initiation Point
UAH	University of Alabama in Huntsville
UAS	Unmanned Aerial System
$V_{\mathrm{f}}, V_{\mathrm{r}}$	downwind (forward) or upwind (reverse) measured velocity of the ADS on a straight flight path
V_{h}^*	steady-state no-wind horizontal velocity of the ADS
V_{v}^*	steady-state no-wind vertical velocity of the ADS
$W, \hat{W}$	horizontal wind speed, true value and estimate
YPG	Yuma Proving Ground

I. Introduction

WIND disturbance during the landing phase of an autonomous parafoil's flight is a leading contributor to miss distance. In fact, flight tests of a small, prototype aerial delivery system (ADS) called *Snowflake* that is being studied jointly by the Naval Postgraduate School and the University of Alabama in Huntsville have indicated that errors due to unknown winds, along with errors in altitude estimation, were the two largest contributors to miss distance.[1]

For an autonomous parafoil system, unknown winds can have their most profound effect on landing accuracy just prior to touchdown. For the Snowflake ADS, during the last 100 m in altitude of the flight, the vehicle will commence a final 180° turn for landing at the target headed directly into the estimated wind; yet, for this final maneuver, the vehicle is relying on a wind estimate that was calculated before the final turn was begun.

The Snowflake algorithm assumes that the final wind estimate is constant down to the target; or, in the case that wind information is being broadcast from a portable weather station at the target, a piecewise constant function is assumed between the last estimated wind speed at altitude, and the measured wind speed at the target.[2]

In contrast to these assumptions, fundamental atmospheric boundary layer theory predicts a logarithmic increase of wind speed with height in the surface layer as in the following relation:

$$W_2 = W_1 \frac{\ln \frac{z_2}{z_0}}{\ln \frac{z_1}{z_0}} \tag{1}$$

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where W_1 and W_2 are wind speeds at heights z_1 and z_2, and parameter z_0 is known as an *aerodynamic roughness length*. The value for the aerodynamic roughness length at the surface is estimated empirically based on the ground terrain. In particular, this parameter has a very small value over the sea surface, and is fairly constant, and therefore easy to estimate.[3] The surface layer is defined as the lower-most region of the atmospheric boundary layer (ABL), from the surface of the Earth to approximately 5% of the boundary layer height.[3] The atmospheric boundary layer is defined as the lower-most portion of the troposphere extending from the surface to as high as 4 km in height. Thus, according to this theory, the functional dependence of wind speed on height can be plotted as a straight line only on a semi-log graph. Figure 1 illustrates this relationship graphically.

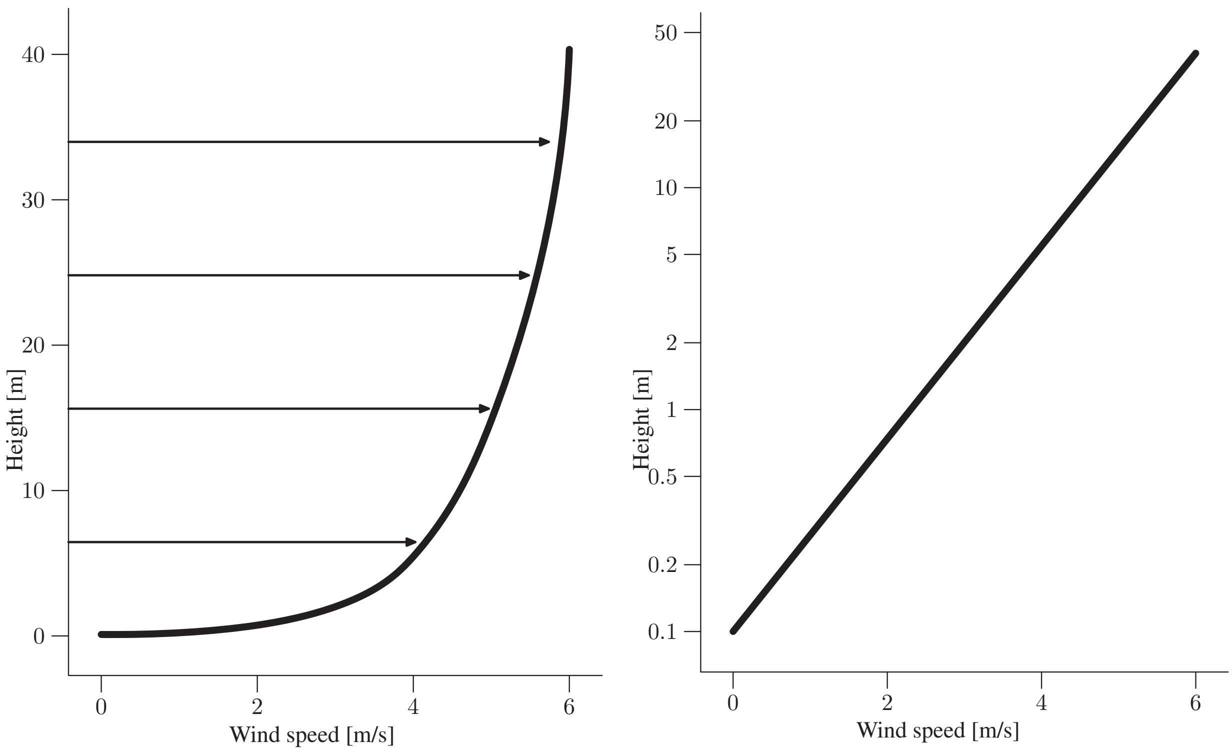

Figure 1. Logarithmic increase of wind speed with height from boundary layer theory.

II. Background

Because there are many experimental and operational ADSs currently undergoing development and testing, different techniques have been developed to mitigate the disturbance caused by the wind on the desired flight path of the ADS. In this paper, specific focus will be on the Snowflake miniature prototype aerial delivery system that was developed through a collaboration between the Naval Postgraduate School and the University of Alabama in Huntsville. An image of Snowflake in flight is shown in Fig. 2, and characteristics of the Snowflake ADS are presented in Table 1. Details of the guidance strategy employed in Snowflake are developed in the following sections.

Table 1. Snowflake miniature aerial delivery system characteristics

Characteristic	Value	Units
dry weight	1.9	kg
canopy span	1.4	m
canopy chord	0.6	m
descent rate	3.7	m/s
forward speed	7.2	m/s
glide ratio	2:1	
minimum turn radius	15.2	m

Figure 2. Small-scale prototype aerial delivery system Snowflake.

A. Aerial Delivery System Trajectory Planning Overview

A survey of existing ADSs was completed in 2006 by Tavan.[4] While there has been much ADS development activity in the subsequent years, a common characteristic of such systems remains: they depend on a preflight wind estimate so that a desired release point relative to the ground target can be computed. This point is called the Computed Air Release Point (CARP) by definition in the U.S. Department of Defense dictionary of terms,[5] and also in one of the first studies to apply the term to autonomous ADSs by Yakimenko *et al.*[6]

Due to the possibly overpowering effect of strong winds on the flight path of the ADS, the CARP is most often located upwind of the ground target so that the ADS has an easier downwind flight to the target. The more sophisticated ADSs such as the NPS/UAH Snowflake plan their trajectory from the CARP to the target in two stages, a loitering stage and an approach stage. During the loitering stage, the Snowflake ADS flies a holding pattern upwind of the target, estimating wind velocity and calculating the moment when it should begin its approach. During the approach stage, the Snowflake flies downwind to a point offset from the ground target, then completes a 180° final approach turn, as detailed in the work by Slegers and Yakimenko.[7] The upwind landing enables a more accurate landing, and reduces the Snowflake's velocity relative to the ground upon impact. A diagram of the overview of the guidance strategy including the loitering pattern is shown in Fig. 3. The loiter pattern is defined by four waypoints at the corners of the box pattern, labeled *A*, *B*, *C*, and *D*. The start of the final 180° turn is known as the *Turn Initiation Point* (TIP), and the distances *away* and *cycle* define respectively the proximity of the loiter pattern to the target, and the major dimension of the loiter pattern.

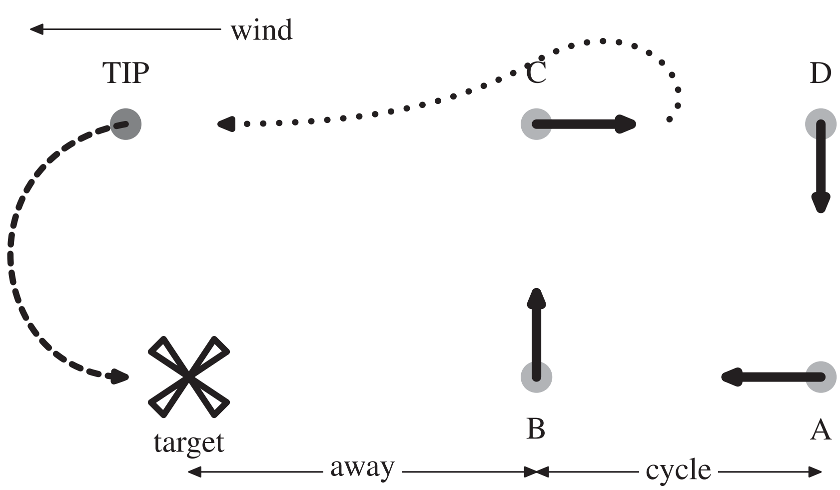

Figure 3. Overview of guidance strategy including loiter pattern.

For analysis and testing, the guidance strategy is divided into phases as summarized below.[7]

Phase 0 After parafoil canopy first opens, no guidance commands are given for a few seconds while the Snowflake's flight path becomes steady.

Phase 1 Snowflake follows heading command to point A.

Phase 2 Snowflake executes waypoint navigation in the loiter pattern A–B–C–D.

Phase 3 Snowflake executes a turn out of the loiter pattern towards the Turn Initiation Point (TIP).

Phase 4 Snowflake lines up to fly directly downwind to the TIP.

Phase 5 Snowflake tracks the optimal turn trajectory towards the target.

Phase 6 Snowflake flies directly upwind along the target line until landing.

B. Wind Estimation in the Snowflake Guidance Algorithm

Any sophisticated ADS that uses a two-stage trajectory plan as described in Section II.A must decide when to switch from the loitering stage to the approach stage. For Snowflake, this decision is made based on a threshold altitude known as z_{start}. In Fig. 3, the transition from the loitering stage to the approach stage is shown by the dotted-line flight path from the loiter pattern towards the TIP. Furthermore, Snowflake must make a second decision about when to begin the final 180° turn: either before or after it has reached abeam the target. The distance from the abeam position to the TIP is given the label D_{switch}; a positive value indicates that the TIP is after the abeam position, and a negative value of D_{switch} indicates that the TIP is before the abeam position. An overview diagram of the terminal guidance maneuver is shown in Fig. 4. Note that the beginning of the flight path in this diagram labeled t_{start} corresponds to the altitude z_{start} at which Snowflake changed from the loitering stage to the approach stage of flight.

t_{start}	time of Snowflake commencing Phase 4 (downwind to TIP)
t_0	time of Snowflake arriving at TIP and commencing Phase 5
t_{exit}	time of Snowflake commencing Phase 6 (steer to target)
t_1	time of Snowflake intercepting final approach course
t_2	time of Snowflake touchdown
T_{downwind}	duration $t_0 - t_{\text{start}}$
T_{turn}	final turn duration $t_1 - t_0$
T_{app}	final approach duration $t_2 - t_1$
D_{switch}	optimal distance to pass the target before commencing final turn
$\tilde{D}$	displacement during final turn due to wind; positive value when point t_1 x-coordinate less than that of t_0
L	distance from target line at t_{start}
L_{app}	straight approach distance from point t_1 to target
R	final turn radius
W	x-component of wind velocity; positive value when blowing toward the origin
$\psi(t)$	reference raw function that is tracked in final turn

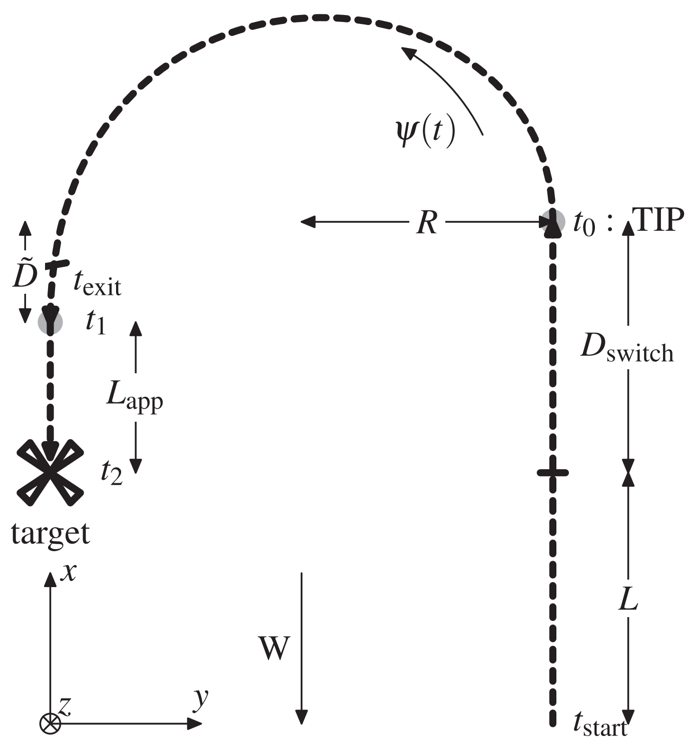

Figure 4. Overview of terminal guidance maneuver.

In summary, the two major decision milestones that the Snowflake ADS guidance algorithm must compute in flight are z_{start} and D_{switch}, in that order. The computation of D_{switch} is the final and most critical decision milestone and is the focus of Section III.A.

Following the derivation by Slegers and Yakimenko,[7] and using a three-dimensional, orthogonal coordinate frame with its origin at the target, and axes as depicted in Fig. 4, expressions are developed for the changes in Snowflake's position in the x and z directions, given known coordinates of the starting and ending positions. In the xz plane, the starting coordinates are $(-L, -z_{\text{start}})$, with L being defined as a positive number, and z_{start} as a negative number. The ending coordinates at the target are $(0,0)$. Summing the starting value with the change in coordinate to equal the ending value both in x and in z, two equations are obtained.

$$-L+L+D_{\text{switch}}+\underbrace{\int_{t_0}^{t_1}\dot{x}\,dt}_{-\tilde{D}}+\underbrace{\int_{t_1}^{t_2}\dot{x}\,dt}_{-L_{\text{app}}}=0 \tag{2}$$

$$z_{\text{start}}+\int_{t_{\text{start}}}^{t_0}\dot{z}\,dt+\int_{t_0}^{t_1}\dot{z}\,dt+\int_{t_1}^{t_2}\dot{z}\,dt=0 \tag{3}$$

Note that Eq. (2) with plus signs before the integral terms is a slight correction to the corresponding equation in Slegers and Yakimenko (Ref. 7, Eq. 35), and that in Eq. (3), z_{start} is a negative number; therefore, all $\dot{z}$ terms are positive (assuming the ADS only descends and never ascends).

In the work of Slegers and Yakimenko,[7] a set of simplifying assumptions is made that enables explicit expressions to be derived for D_{switch} and z_{start}. One of these simplifying assumptions is that the wind profile, or the wind speed over a range of altitudes, is constant or piecewise constant. Examples of a constant and a piecewise constant wind profile are shown in Fig. 5. Note that for these plots of wind profile, the independent variable, altitude z, is plotted on the vertical axis.

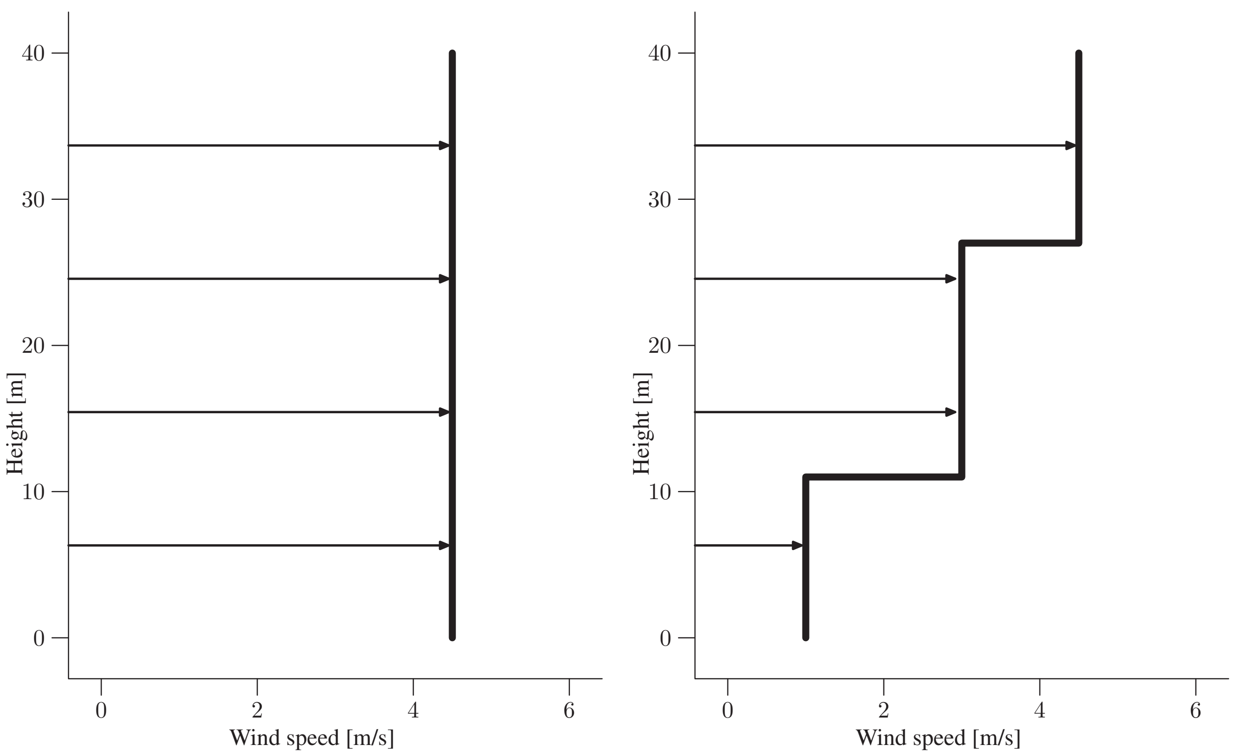

Figure 5. Examples of constant and piecewise-constant wind profiles.

The method used by the Snowflake ADS in flight to estimate wind speed is very simple and does not require the use of a pitot-static airspeed sensor. An *a priori* estimate of the wind direction is used to align the loiter pattern so that the long dimension of the rectangle *ABCD* is in the direction of the wind estimate. The result is that the two long segments of the rectangular loiter pattern should be aligned very nearly directly downwind and upwind. Referring to Fig. 3, the *forward* velocity that will be measured by the on-board GPS receiver as the Snowflake travels in the direction of *A* to *B* on the downwind segment will be

$$V_{\text{f}}=V_{\text{h}}^{*}+W \tag{4}$$

and, similarly, the *reverse* velocity measured as the Snowflake travels in the direction of *C* to *D* on the upwind segment will be

$$V_{\text{r}}=V_{\text{h}}^{*}-W \tag{5}$$

Using Eq. (4) and Eq. (5), including speed measurements from both the downwind and upwind segments, an estimate of the wind $\hat{W}$ can be computed with

$$\hat{W} = \frac{V_{\mathrm{f}} - V_{\mathrm{r}}}{2} \tag{6}$$

In practice, for each execution of the Snowflake main program loop, an estimate of the wind is calculated using Eq. (6) and the most recently measured values of the velocities V_{f} and V_{r}.

III. Analysis

The assumption of a constant or piece-wise constant wind profile made in Section II.B is an extreme simplification of a very complex and variable process. The assumption of a constant profile, while yielding a mathematical problem that is much more tractable, is an assumption that has no basis in meteorological boundary layer theory. Instead, meteorological theory postulates that within the ABL, whose height varies between 200 m and 4 km from the Earth's surface, there exists a lower layer within which heat and moisture interactions between the atmosphere and the Earth's surface cause significant changes in wind speed with height. This lower layer extends from the surface of the Earth up to a height between approximately 20 m to 200 m and is known as the surface layer.[3]

The work by Stull[3] presents three cases of overall atmospheric conditions, and three corresponding mathematical models for the general wind profiles under these conditions. The simplest wind profile, the logarithmic model, corresponds to the case of neutral atmospheric stability. Neutral stability refers to the tendency of a parcel of air, once displaced vertically from its original position, to remain in its new position, neither continuing vertical motion, nor returning to its original position. It will be assumed in this paper that the logarithmic model corresponding to neutral surface layer atmospheric stability, prevalent on overcast and windy days, is the appropriate wind profile model to implement into the Snowflake ADS.

It is noteworthy that a couple of different research efforts have set forth theoretical bases for dealing with wind models containing arbitrary wind profiles;[8,9] however, in this paper, the proposed logarithmic wind model will be compared only to the previously implemented constant wind model. The extension of comparison of the logarithmic wind model as well as the constant wind model to some of the newly-developed wind models is a worthy subject of future research.

A. Iterative Calculation of Dswitch Using a Logarithmic Wind Profile

In the following derivation, the assumption of a constant or piecewise constant wind profile will be discarded and replaced by the assumption that wind speed W varies logarithmically with altitude z as:

$$W(z) = \alpha \ln(-z) + \beta \tag{7}$$

where α and β are constants. Note that, since altitude z is represented as a negative number according to the sign convention in use, an additional factor of -1 is included in the argument of the logarithm. The derivation uses this assumption for the wind profile, along with the assumptions that the steady-state, no-wind values of the ADS horizontal and vertical velocities are known and labeled V_{h}^* and V_{v}^*. Furthermore, it is assumed that the wind vector lies only in the horizontal plane, and is parallel with the x axis of the coordinate system depicted in Fig. 4. Using the assumptions of constant steady-state no-wind vertical velocity and no vertical wind component, the time derivative of the z coordinate, $\dot{z}$, is just equal to V_{v}^*, where downward change has a positive sign. Thus, the integral terms in Eq. (3) can be simplified.

$$z_{\mathrm{start}} + V_{\mathrm{v}}^* T_{\mathrm{downwind}} + V_{\mathrm{v}}^* T_{\mathrm{turn}} + V_{\mathrm{v}}^* T_{\mathrm{app}} = 0 \tag{8}$$

where T_{downwind} represents the time duration required to travel the sum of distances L and D_{switch}. Again, recall that under the sign convention being used here, z_{start} is a negative number and that V_{v}^* and all time durations are positive. Assuming that the Snowflake's turn rate is constant, and that the turn will encompass 180° of a circle of known radius R, the time duration for the turn T_{turn} is a known constant in terms of R and V_{h}^* (see also Ref. 7 Eq. 33). Therefore, this equation has only two unknowns: T_{downwind} and T_{app}.

Under the assumption of a constant wind profile, a simple expression for T_{downwind} could be written in terms of unknown value D_{switch}. Also using the constant wind profile assumption, Eq. (2) simplified into an equation containing two unknowns: D_{switch} and T_{app}; therefore the two linear equations in two unknowns could easily be solved. Under the assumption of a logarithmic wind profile as in Eq. (7), T_{downwind} cannot be replaced by a simple expression containing D_{switch}. Instead, in order to compute T_{downwind} assuming that the wind profile is logarithmic, a value of D_{switch} must be assumed, and then the time t_0 can be computed. Time t_0 is computed using the assumption of constant vertical velocity from the following simple relation:

$$t_0 = \frac{z_0 - z_{\text{start}}}{V_{\text{v}}^*} + t_{\text{start}} \tag{9}$$

where altitude z_0 had been calculated using the Lambert W function as follows:

$$z_0 = \frac{c}{bW\left(-\frac{ce^{a/b}}{b}\right)} \tag{10}$$

where a, b, and c are constants that can be expressed in terms of known quantities V_{h}^*, V_{v}^*, α, β, L, z_{start}, and assumed quantity D_{switch}. See the appendix for the derivation of Eq. (10).

The iteration procedure to calculate D_{switch} is as follows:

1. Select an initial guess value for D_{switch}
2. Using the initial guess value for D_{switch}, calculate z_0 using Eq. (10), and T_{downwind} from:

$$T_{\text{downwind}} = \frac{z_0 - z_{\text{start}}}{V_{\text{v}}^*} \tag{11}$$

3. Using T_{downwind}, calculate T_{app} using Eq. (8)

$$T_{\text{app}} = \frac{-z_{\text{start}}}{V_{\text{v}}^*} - T_{\text{turn}} - T_{\text{downwind}} \tag{12}$$

4. Compute z_1 using the assumption of constant vertical velocity

$$z_1 = z_0 + V_{\text{v}}^* T_{\text{turn}} \tag{13}$$

5. Compute $\tilde{D}$ from known quantities:

$$\tilde{D} = (\beta - \alpha)T_{\text{turn}} - \frac{\alpha}{V_{\text{v}}^*}\left[-z_0 \ln(-z_0) - z_1 \ln(-z_1)\right] \tag{14}$$

   See the appendix for the derivation of Eq. (14).

6. Compute L_{app} using T_{app} using:

$$L_{\text{app}} = (V_{\text{h}}^* - \alpha + \beta)T_{\text{app}} + \frac{\alpha e^{-\beta/\alpha}}{V_{\text{v}}^*} + \alpha T_{\text{app}} \ln(T_{\text{app}} V_{\text{v}}^*) \tag{15}$$

   See the appendix for the derivation of Eq. (15).

7. Compute D_{switch} from:

$$D_{\text{switch}} = \tilde{D} + L_{\text{app}} \tag{16}$$

8. Use value of D_{switch} computed in step 7 as the next iteration estimate of D_{switch} in step 1. Repeat iterations until differences between subsequent values of D_{switch} are below a threshold value.

B. Adaptive Filtering for Parameter Estimation

In Section III.A, the parameters of the logarithmic wind profile, α and β, were treated as known parameters. These two parameters are estimated from the output of an adaptive filtering algorithm, which is shown in block diagram form in Fig. 6. In this representation, the logarithmic wind model is interpreted in a discrete-time form with desired signal $d[n]$ containing the sequential estimates of wind speed from the Snowflake guidance algorithm. These sequential estimates are obtained using Eq. (6) as described in Section II.B. When converting the logarithmic wind profile model in Eq. (7) into discrete-time form for the adaptive filtering algorithm, then measurement vector $H[n]$ contains the natural logarithm of the current altitude measurement $z[n]$. The estimate of wind speed $\hat{d}[n]$ at any altitude $z[n]$ is given by the following product:

$$\hat{d}[n] = \underbrace{\begin{bmatrix}\ln(-z[n]) & 1\end{bmatrix}}_{H[n]} \underbrace{\begin{bmatrix}\alpha \\ \beta\end{bmatrix}}_{\hat{\Theta}[n]} \tag{17}$$

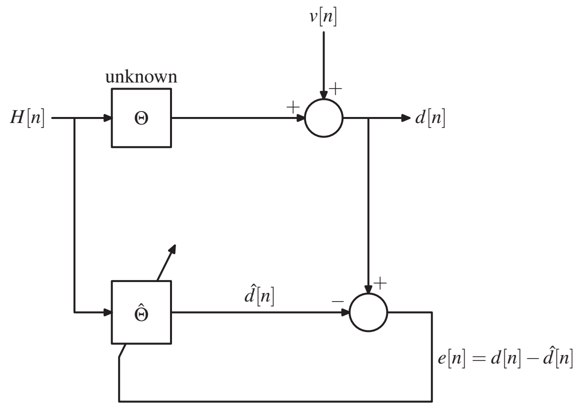

Figure 6. Block diagram of Recursive Least Squares adaptive filtering algorithm.

The term on the right side of Eq. (17) can be represented as the product of the measurement matrix $H[n]$ (a row vector in this case) and an estimated parameter vector $\hat{\Theta}[n]$. Note that the "true" parameter vector $\Theta[n]$ remains unknown. In the adaptive filtering model, the true output (desired signal $d[n]$) is known; however, the "true" logarithmic wind profile parameters and the measurement noise signal $v[n]$ that compose the true wind speed are unknown.

The Recursive Least Squares (RLS) adaptive filtering algorithm can be applied to this parameter estimation problem; various digital signal processing textbooks contain the derivation of this algorithm.[10,11] In summary, this algorithm minimizes the total error given by the following sum of weighted error squares:

$$\Xi[n] = \sum_{i=1}^{n} \lambda^{n-i} |e[i]|^2 = \sum_{i=1}^{n} \lambda^{n-i} \left| d[i] - H[i]\hat{\Theta}[n] \right|^2 \tag{18}$$

where λ is known as the *forgetting factor* and typically has a value equal to or greater than 0.9.[10] Therefore, the few most recent values of the *a posteriori* error signal $e[n]$ are the most influential in the calculation of total error $\Xi[n]$. As each new value of the measurement matrix $H[n]$ and the desired output signal $d[n]$ is obtained, the new values of the estimated parameters $\hat{\Theta}[n]$ are calculated by adding to the previous estimate $\hat{\Theta}[n-1]$ a quantity proportional to the *a priori* error value $\xi[n]$. The *a priori* error is formed by using the previous parameter estimate values $\hat{\Theta}[n-1]$ with current measurement matrix $H[n]$ and subtracting the product from the current desired signal value $d[n]$. The constant or proportionality in this operation is known as the adaptation gain vector $\mathbf{K}[n]$, which depends in turn on the measurement matrix $H[n]$ and a 2×2 matrix (for the current problem) known as the *inverse correlation matrix* $\mathbf{P}[n]$. The Recursive Least Squares adaptive filtering algorithm is summarized as follows:

1. Initialize the inverse correlation matrix $\mathbf{P}(0)$ using the identity matrix $\mathbf{I}$ and a small positive constant δ.

$$\mathbf{P}(0) = \delta \mathbf{I} \tag{19}$$

2. Compute the adaptation gain vector $\mathbf{K}[n]$

$$\mathbf{K}[n] = \frac{\lambda^{-1}\mathbf{P}[n-1]H^T[n]}{1 + \lambda^{-1}H[n]\mathbf{P}[n-1]H^T[n]} \tag{20}$$

3. Compute the *a priori* error value $\xi[n]$

$$\xi[n] = y[n] - H[n]\hat{\Theta}[n-1] \tag{21}$$

4. Update the estimate of the parameters of the wind model $\hat{\Theta}[n]$

$$\hat{\Theta}[n] = \hat{\Theta}[n-1] + \mathbf{K}[n]\xi[n] \tag{22}$$

5. Update the value of the inverse correlation matrix $\mathbf{P}[n]$

$$\mathbf{P}[n] = \lambda^{-1}\mathbf{P}[n-1] - \lambda^{-1}\mathbf{K}[n]H[n]\mathbf{P}[n-1] \tag{23}$$

6. Increment discrete counter n, obtain new values of desired signal $d[n]$, and altitude $z[n]$ (which is used in forming the one-dimensional measurement matrix $H[n]$) and then repeat starting at step 2.

IV. Algorithm Implementation

Up to this point, a method for Snowflake to use a sequence of altitude and velocity measurements in order to estimate the parameters of a logarithmic wind profile has been proposed in Section III.B, and a means to use those parameters in the calculation of decision milestone D_{switch} has been described in Section III.A. In the sequel, the incorporation of these new methods into the existing code for the Snowflake ADS guidance algorithm will be described, along with the implications of these changes.

Section II.B listed the two major decision milestones to be computed in flight as transition altitude z_{start} and transition distance D_{switch}. As also mentioned in that section, of the two decision milestones, D_{switch} was the final and more critical of the two. In the following algorithm, the estimated parameters of the logarithmic wind profile are applied to the calculation of only D_{switch} for the additional reason that the transition altitude z_{start} is typically at an altitude above the limit of where the surface layer logarithmic wind profile assumption is considered to be valid (approximately up to a maximum of 200 m).[3]

The incorporation of the algorithm into the Snowflake ADS code would be as follows:

1. At each execution of the main loop, measure current altitude $z[n]$ and, if on a downwind or upwind segment, measure V_{f} or V_{r} as appropriate.

2. If altitude is below a threshold value, for instance 200 m above ground level (AGL), and Snowflake is in phase 4, use adaptive filter algorithm as described in Section III.B in order to estimate parameters α and β of logarithmic wind profile. If not yet in phase 4, return to step 1.

3. Use current values of parameters α and β in iterative computation as described in Section III.A to compute D_{switch}.

4. Compare Snowflake's current x coordinate (as defined in Fig. 4). If $x < D_{\text{switch}}$, return to step 1.

5. If $x > D_{\text{switch}}$, then begin phase 5 and start final turn to target. Note that the comparison of x to D_{switch} is valid whether D_{switch} is negative or positive.

It should be seen in simulation that a Snowflake relying on the logarithmic wind profile should, all other aspects being equal, begin the final turn towards the target later than a Snowflake relying on the assumption of a constant wind profile because the logarithmic wind profile should predict smaller values of wind speed near the surface.

V. Simulation Results

The algorithm detailed in Section IV for applying a logarithmic wind model to horizontal wind estimates generated in flight was first applied to recorded Snowflake telemetry data from previous flight tests. The data set from flight tests at Yuma Proving Ground, Yuma, Arizona, in October 2008[1] was chosen for processing first due to the existence of rawinsonde data concurrent with the Snowflake flights. The recorded Snowflake horizontal wind estimates were processed using the RLS algorithm in order to determine what the logarithmic wind model parameters α and β would have been had they been calculated in flight. Because Snowflake's recorded wind estimates are no longer updated once the Snowflake commences phase 5 (final turn to target,) the last values of α and β calculated were used to generate a logarithmic wind profile that was then compared to the rawinsonde data at altitudes corresponding to those of the Snowflake during final approach. Figure 7 illustrates such a comparison.

In Fig. 7, data is presented from drop number 12 on October 21, 2008 from a test series conducted at Yuma Proving Ground, Arizona.[1] The post-processed calculation of D_{switch} indicates that the Snowflake would have commenced the final turn later than was done using the constant wind assumption, which would have been a correct decision since the original flown trajectory resulted in overshoot, as is depicted in Fig. 8.

The next phase of simulation was conducted in conjunction with flight tests conducted at Camp Roberts, California, 21 to 24 February, 2011. These trials are described as simulations, because, while the Snowflake was indeed generating horizontal wind speed estimates in flight and using those to calculate logarithmic wind model parameters α and β, the Snowflakes were not using this information to determine a value for D_{switch} based on the logarithmic wind profile assumption. Rather, the final turn decision for the Snowflakes in these trials was based upon the constant wind profile assumption.

Figure 9 shows the estimated logarithmic wind profile generated in post-flight simulation using the wind estimates gathered in flight from drop number 6 on February 24, 2011. The value of D_{switch} computed using the logarithmic wind profile is compared to the actual turn start point computed assuming a constant wind profile, and the turn using the logarithmic wind profile would have occurred later.

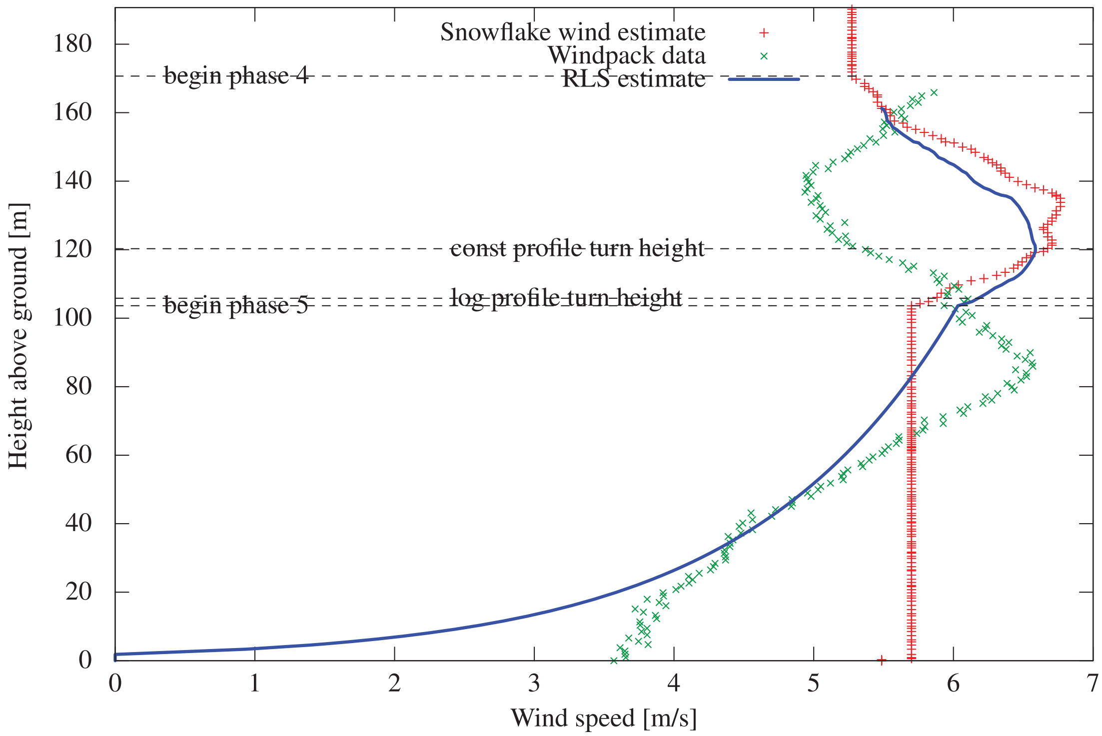

Figure 7. Comparison of D_{switch} calculated using logarithmic wind profile, and using constant wind profile from October 2008 data.

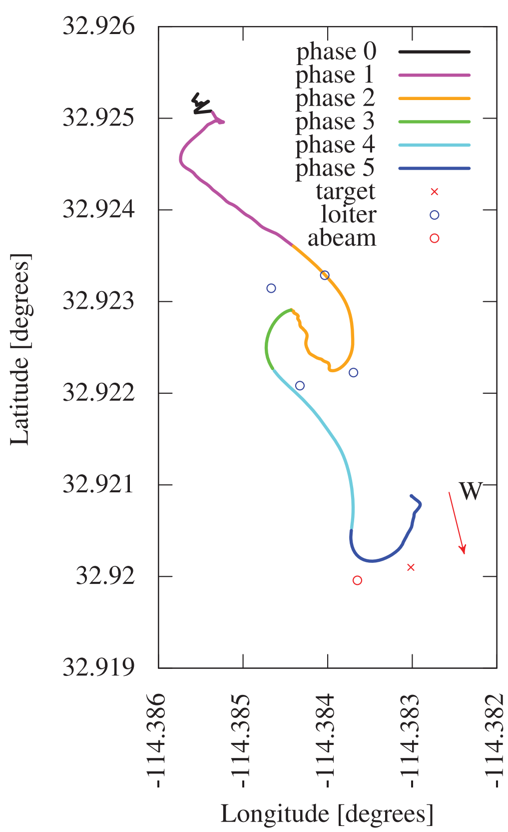

Figure 8. Trajectory of Snowflake flown using constant wind profile from October 2008 data.

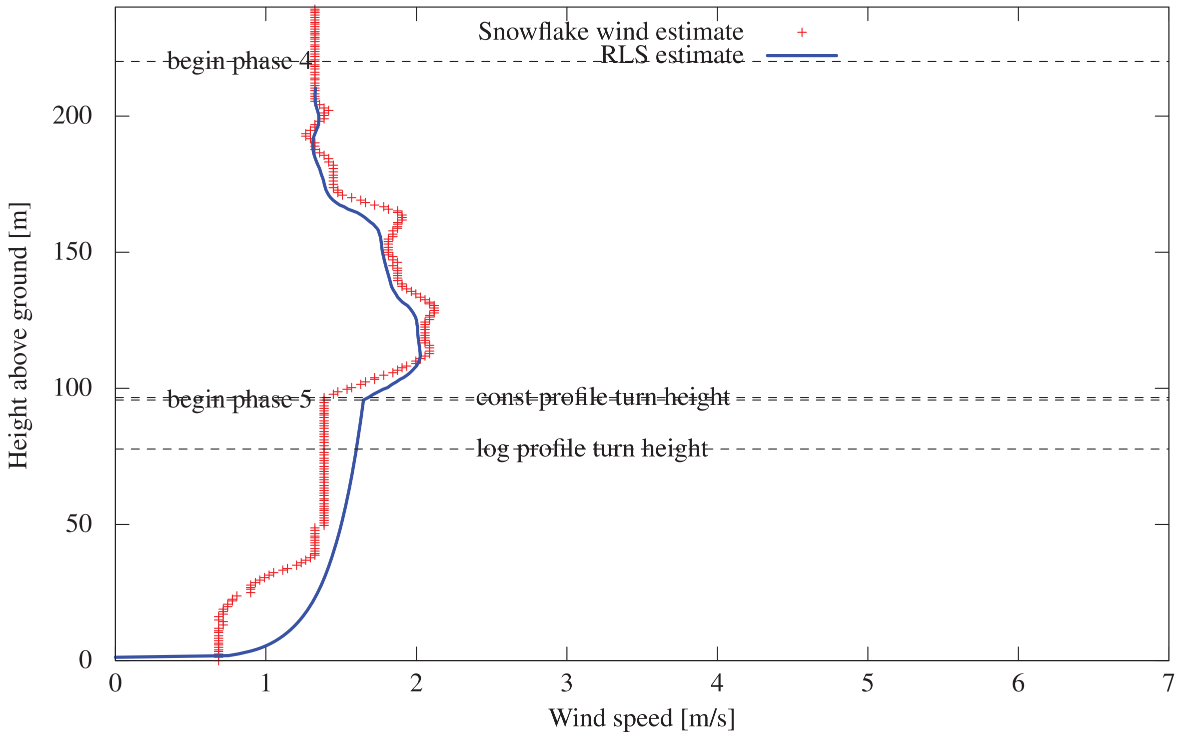

Figure 9. Comparison of Snowflake constant wind profile estimate to RLS logarithmic wind profile.

VI. Flight Test Results

The full implementation of the adaptive filtering algorithm as described in Section IV, including the calculation of D_{switch} was subsequently incorporated into the source code for the autopilot installed in the Snowflake ADS. The systems thus modified were tested in flight after being dropped from an altitude of 762 m to 914 m (2500 to 3000 ft) AGL by an Arcturus T-20 unmanned aerial system (UAS) over McMillan Airfield (identifier CA62) at Camp Roberts, California, in a series of tests during 2 to 4 May, 2011. In order to achieve a good comparison between the logarithmic wind profile assumption and the constant wind profile assumption, on each flight of the T-20, the two Snowflakes were dropped, one from underneath each wing, as quickly as the two under-wing release mechanisms could be opened. Usually, both Snowflakes were dropped within one second of each other.

Figure 10 shows the in-flight wind estimate calculated by the Snowflake from drop number 2 on May 3, 2011, using the logarithmic wind profile model. As the Snowflake begins phase 5 (final approach turn) and phase 6 (straight flight to target), the wind profile estimate produced by the final calculated values of α and β is shown along with the actual wind estimates produced during phase 6. From these two plots, it can be inferred that the logarithmic model agrees reasonably well with the measured data.

Figure 11 depicts the in-flight estimates made by a Snowflake in flight of the two parameters L_x and D_{switch}. Recall that the first parameter L_x is the current distance from the Snowflake to a position abeam of the target, as defined in Fig. 4; this parameter is defined as positive when the Snowflake has not yet reached the abeam position, and negative after the Snowflake has flown past the abeam position. Because D_{switch} is defined such that positive values represent a turn past the abeam position, and negative values indicate a turn prior to the abeam position, the criterion upon which the Snowflake algorithm commences the final turn is $L_x < -D_{\text{switch}}$. Figure 11 shows that once one Snowflake began calculating D_{switch} using the logarithmic wind profile model, its D_{switch} estimate went from negative (turn prior to the target) to positive (turn after abeam the target). The other Snowflake, relying on the constant wind profile model did begin its final turn prior to the target.

Figures 12 and 13 show flight trajectories of both of these Snowflakes. While the Snowflake that used the constant wind profile model did land closer to the target, its trajectory was less regular, since it had to enter an overhead spiraling approach to the target after flying over the target too high due to its early turn. For potential shipboard landing of an ADS, a predictable, non-spiraling approach trajectory is better.

VII. Conclusion

The simulation and flight test results have provided evidence that the logarithmic model of the surface layer wind profile is a usable and useful model that can aid proper trajectory planning by an ADS in the terminal approach phase.

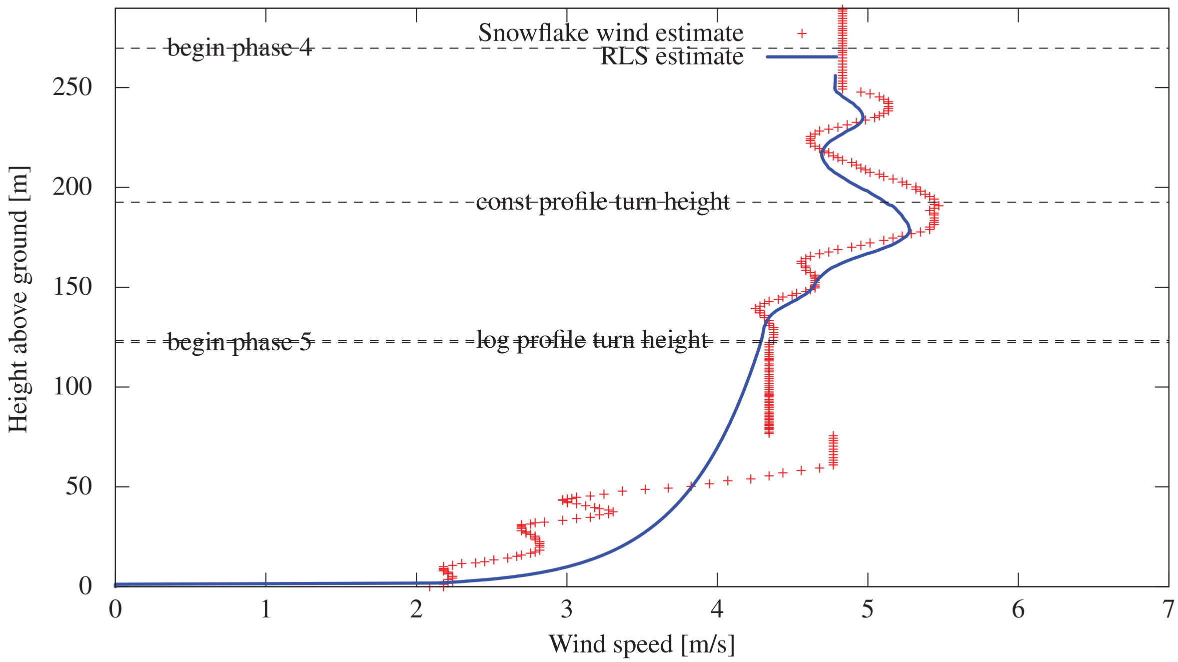

Figure 10. Estimated wind profile using the logarithmic wind model

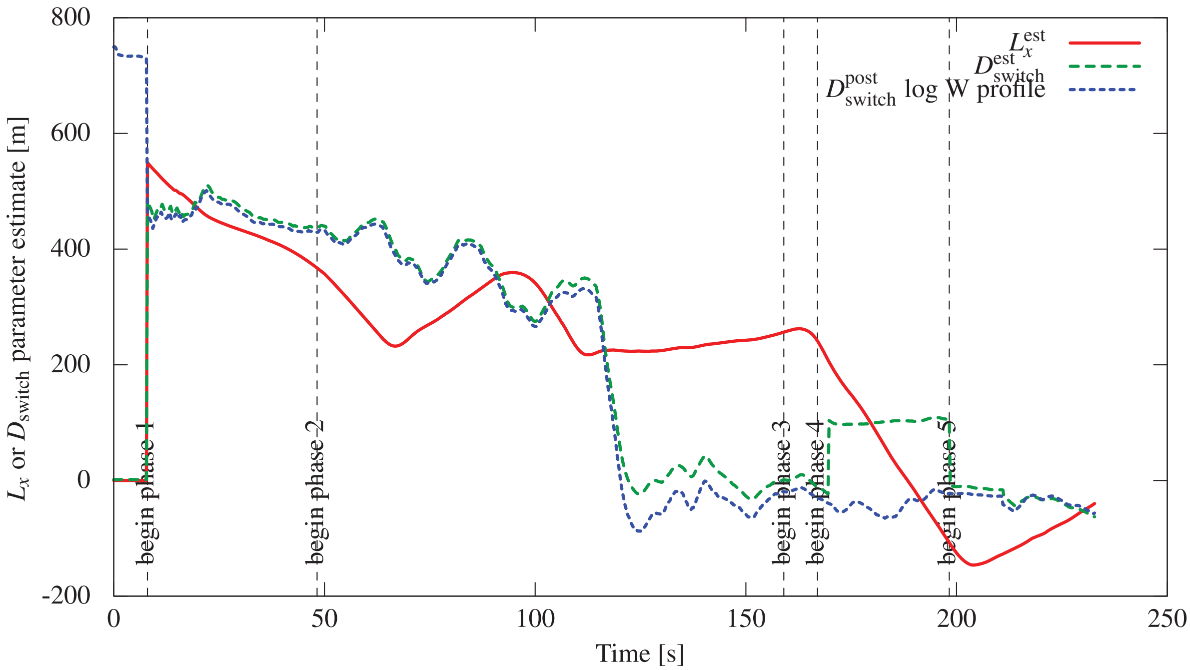

Figure 11. L_x **and** D_{switch} **in-flight estimates from Snowflake using logarithmic wind model.**

Features of the logarithmic wind model that make it especially applicable to the maritime domain are two-fold. First, use of the logarithmic wind model has the potential to reduce instances of beginning the final turn early, avoiding an overhead spiraling approach to the target, and leading to a standard, straight-in approach more conducive to shipboard landing. Second, the required *a priori* parameter of the logarithmic wind model, the aerodynamic roughness length, is a parameter that is more easily estimated at sea than in most terrain on land because it has such a small value over the open sea.

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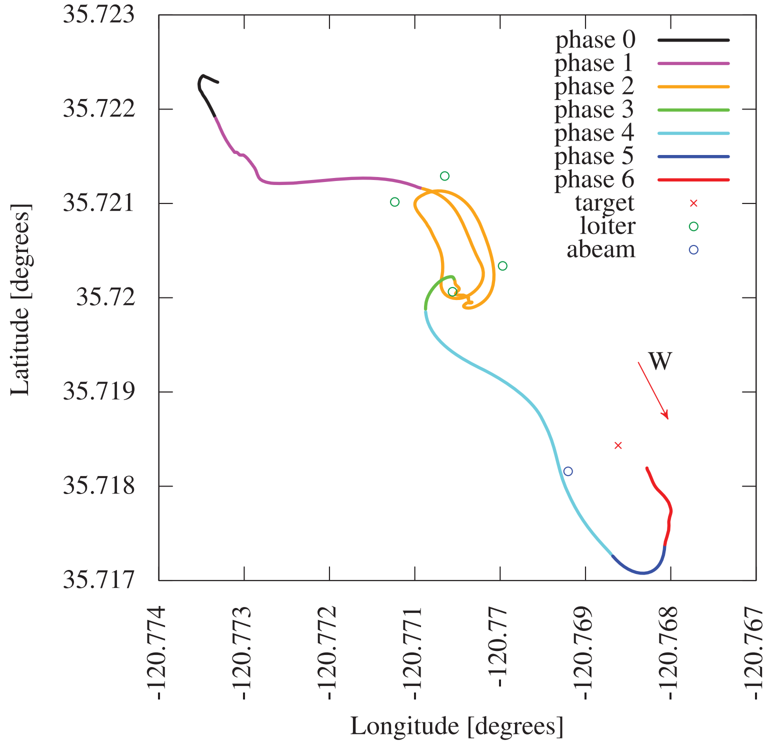

Figure 12. Flight trajectory for Snowflake using logarithmic wind profile model.

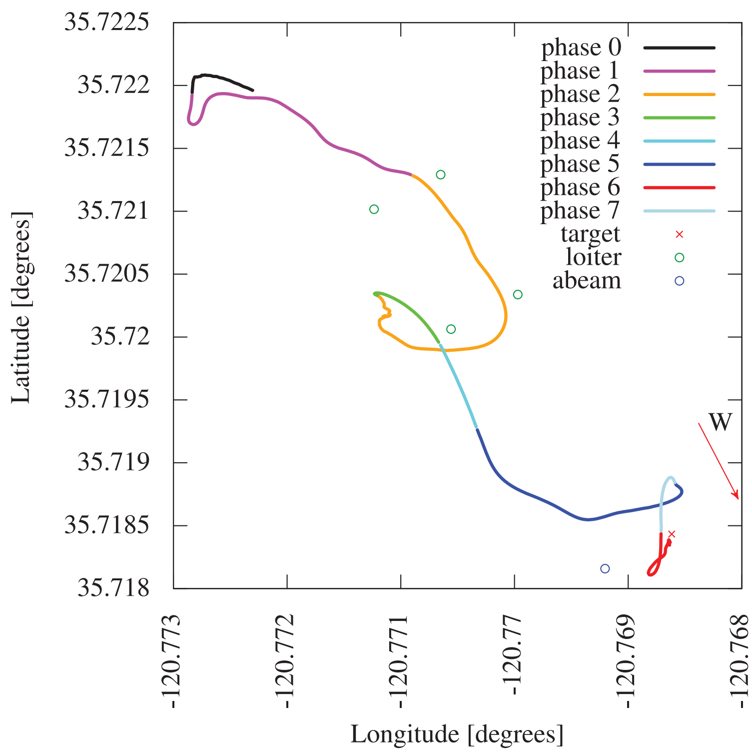

Figure 13. Flight trajectory for Snowflake using constant wind profile model.

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Appendix: Derivation of Solutions for Altitude z_0 and Distances $\tilde{D}$ and L_{app}

Derivation of Solution for Altitude z_0

In order to compute z_0, start with the fundamental expression for movement in the x direction between times t_0 and t_{start}, and resolve motion into a component that is due to no-wind velocity, and a component due to the wind. This expression will contain unknown value time t_0 instead of the corresponding altitude at that time, z_0; however, there exists a simple variable transformation from time to altitude based on the assumption of constant descent rate.

$$\int_{t_{\text{start}}}^{t_0} \dot{x}\, dt = L + D_{\text{switch}} \tag{24}$$

$$V_{\text{h}}^{*}(t_0 - t_{\text{start}}) - \int_{t_{\text{start}}}^{t_0} W(z)\, dt = L + D_{\text{switch}} \tag{25}$$

Perform a variable transformation of both terms, converting from time to altitude as the independent variable. Altitudes corresponding to t_{start} and t_0 are labeled z_{start} and z_0 respectively.

$$z(t) = V_{\text{v}}^{*}(t - t_{\text{start}}) + z_{\text{start}} \tag{26}$$

$$dz = V_{\text{v}}^{*} dt \tag{27}$$

$$z(t_0) \equiv z_0 = V_{\text{v}}^{*}(t_0 - t_{\text{start}}) + z_{\text{start}} \Rightarrow t_0 - t_{\text{start}} = \frac{z_0 - z_{\text{start}}}{V_{\text{v}}^{*}} \tag{28}$$

$$\frac{V_{\text{h}}^{*}}{V_{\text{v}}^{*}}(z_0 - z_{\text{start}}) - \frac{1}{V_{\text{v}}^{*}} \int_{z_{\text{start}}}^{z_0} W(z)\, dz = L + D_{\text{switch}} \tag{29}$$

Move known quantity involving z_{start} to the right side, and substitute in Eq. (7) on page 6 for logarithmic wind profile.

$$\frac{V_{\text{h}}^{*}}{V_{\text{v}}^{*}} z_0 - \frac{1}{V_{\text{v}}^{*}} \int_{z_{\text{start}}}^{z_0} \alpha \ln(-z) + \beta\, dz = L + D_{\text{switch}} + \frac{V_{\text{h}}^{*}}{V_{\text{v}}^{*}} z_{\text{start}} \tag{30}$$

Perform variable substitution to account for the fact that altitude is represented as a negative number in the sign convention in use.

$$\begin{aligned} z' &= -z \\ dz' &= -dz \end{aligned} \tag{31}$$

$$\frac{V_{\text{h}}^{*}}{V_{\text{v}}^{*}} z_0 + \frac{1}{V_{\text{v}}^{*}} \int_{-z_{\text{start}}}^{-z_0} \alpha \ln z' + \beta\, dz' = L + D_{\text{switch}} + \frac{V_{\text{h}}^{*}}{V_{\text{v}}^{*}} z_{\text{start}} \tag{32}$$

Evaluate the definite integral.

$$\frac{V_\mathrm{h}^*}{V_\mathrm{v}^*} z_0 + \frac{\beta}{V_\mathrm{v}^*}(-z_0 + z_\mathrm{start}) + \frac{\alpha}{V_\mathrm{v}^*}\left[z' \ln z' - z'\right]_{-z_\mathrm{start}}^{-z_0} = L + D_\mathrm{switch} + \frac{V_\mathrm{h}^*}{V_\mathrm{v}^*} z_\mathrm{start} \tag{33}$$

Evaluate the limits of integration, and move all known quantities to the right side, assuming D_switch is a known quantity.

$$\left(\frac{V_\mathrm{h}^* - \beta}{V_\mathrm{v}^*}\right) z_0 + \frac{\alpha}{V_\mathrm{v}^*}\left[-z_0 \ln(-z_0) + z_0 + z_\mathrm{start} \ln(-z_\mathrm{start}) - z_\mathrm{start}\right] = L + D_\mathrm{switch} + \left(\frac{V_\mathrm{h}^* - \beta}{V_\mathrm{v}^*}\right) z_\mathrm{start} \tag{34}$$

$$\left(\frac{V_\mathrm{h}^* + \alpha - \beta}{V_\mathrm{v}^*}\right) z_0 - \frac{\alpha}{V_\mathrm{v}^*} z_0 \ln(-z_0) = L + D_\mathrm{switch} + \left(\frac{V_\mathrm{h}^* + \alpha - \beta}{V_\mathrm{v}^*}\right) z_\mathrm{start} - \frac{\alpha}{V_\mathrm{v}^*} z_\mathrm{start} \ln(-z_\mathrm{start}) \tag{35}$$

Equation Eq. (35) is an equation in only one unknown, z_0, and has the form

$$a z_0 + b z_0 \ln(-z_0) = c \tag{36}$$

and has a solution in terms of the Lambert W function of:

$$z_0 = \frac{c}{bW\left(-\frac{ce^{a/b}}{b}\right)} \tag{37}$$

where the Lambert W function is defined in the work by Corless *et al.*[12] to be the function that satisfies:

$$W(z)e^{W(z)} = z \tag{38}$$

Derivation of Solution for Distance $\tilde{D}$

In order to compute $\tilde{D}$, start with an expression for movement in the x direction between times t_0 and t_1. The sign convention established by Slegers and Yakimenko[7] is that the distance $\tilde{D}$ is measured in the negative x direction from point t_1. In other words, if point t_1 has an x coordinate that is less than that of point t_0, then distance $\tilde{D}$ is said to have a positive value. Put in yet another way, if, during the turn from point t_0 to point t_1, the wind pushes Snowflake closer to the target, then $\tilde{D}$ is said to have a positive value; conversely, if the wind pushes Snowflake further away from the target during the turn, then $\tilde{D}$ is said to have a negative value. Therefore, $\tilde{D}$ has the opposite sign of the change in x coordinate between times t_0 and t_1.

$$\tilde{D} = -\int_{t_0}^{t_1} \dot{x}\, dt \tag{39}$$

For the next step, it is assumed that the turn trajectory from t_0 to t_1 is planned in such a way that, if there were no wind, that point t_1 would have the same x coordinate value as point t_0. Therefore, any deviation, measured as $\tilde{D}$, is a result of the wind alone. Noting that the sign convention for wind in Fig. 4 on page 4 is such that positive values of wind W correspond to negative values of resultant movement in the x direction, or $\dot{x}$, Eq. (39) on this page is restated as

$$\tilde{D} = \int_{t_0}^{t_1} W(z)\, dt \tag{40}$$

Perform a variable transformation as in Eq. (26) on the preceding page on the integral term, converting from time to altitude as the independent variable. Altitudes corresponding to t_0 and t_1 are labeled z_0 and z_1 respectively.

$$\tilde{D} = \frac{1}{V_\mathrm{v}^*}\int_{z_0}^{z_1} W(z)\, dz \tag{41}$$

Substitute in Eq. (7) on page 6 for logarithmic wind profile.

$$\tilde{D} = \frac{1}{V_\mathrm{v}^*}\int_{z_0}^{z_1} \alpha \ln(-z) + \beta\, dz \tag{42}$$

Perform variable substitution as in Eq. (31) on page 14 to account for the fact that altitude is represented as a negative number in the sign convention in use.

$$\tilde{D} = -\frac{1}{V_{\mathrm{v}}^*} \int_{-z_0}^{-z_1} \alpha \ln z' + \beta \, dz' \tag{43}$$

Evaluate the definite integral and the limits of integration and group terms.

$$\tilde{D} = \frac{\beta}{V_{\mathrm{v}}^*}(z_1 - z_0) - \frac{\alpha}{V_{\mathrm{v}}^*}\Big[z' \ln z' - z'\Big]_{-z_0}^{-z_1} \tag{44}$$

$$\tilde{D} = \frac{\beta}{V_{\mathrm{v}}^*}(z_1 - z_0) - \frac{\alpha}{V_{\mathrm{v}}^*}\Big[-z_1 \ln(-z_1) + z_1 + z_0 \ln(-z_0) - z_0\Big] \tag{45}$$

$$\tilde{D} = \frac{\beta}{V_{\mathrm{v}}^*}(z_1 - z_0) - \frac{\alpha}{V_{\mathrm{v}}^*}(z_1 - z_0) - \frac{\alpha}{V_{\mathrm{v}}^*}\Big[z_0 \ln(-z_0) - z_1 \ln(-z_1)\Big] \tag{46}$$

Altitude is again related to time using Eq. (26) on page 14, yielding the following relations:

$$z(t_1) \equiv z_1 = V_{\mathrm{v}}^*(t_1 - t_{\mathrm{start}}) + z_{\mathrm{start}} \tag{47}$$

$$z_1 - z_0 = V_{\mathrm{v}}^*(t_1 - t_0) \equiv V_{\mathrm{v}}^* T_{\mathrm{turn}} \tag{48}$$

Using Eq. (48) on the current page, a final expression is obtained that includes values that are known *a priori*: α, β, T_{turn}, and V_{v}^*. This expression also includes altitude values z_0 and z_1 at the beginning and the end of the turning maneuver. Once one of these altitude values is determined or estimated, the other is easily found using the known time for the turning maneuver, T_{turn} and the assumed constant descent rate V_{v}^*.

$$\tilde{D} = (\beta - \alpha) T_{\mathrm{turn}} - \frac{\alpha}{V_{\mathrm{v}}^*}\Big[z_0 \ln(-z_0) - z_1 \ln(-z_1)\Big] \tag{49}$$

Derivation for Solution of Distance L_{app}

In order to compute distance L_{app}, start with the fundamental expression for movement in the x direction between times t_1 and t_2, which is the time of landing. The distance L_{app} is defined as a positive distance; however, the change in x coordinate between times t_1 and t_2 is negative. Therefore, L_{app} will have the opposite sign from the integral of $\dot{x}$.

$$L_{\mathrm{app}} = -1 \cdot \int_{t_1}^{t_2} \dot{x} \, dt \tag{50}$$

The motion is then resolved into a component that is due to no-wind velocity, and a component due to the wind. During this segment of the flight path, the no-wind velocity V_{h}^* and motion due to positive values of wind W are both assumed to be aligned in the opposite direction of positive values of $\dot{x}$; therefore a negative sign is inserted into the integrand.

$$L_{\mathrm{app}} = -1 \cdot \int_{t_1}^{t_2} -(V_{\mathrm{h}}^* + W(z)) \, dt \tag{51}$$

Next, substitute in Eq. (7) on page 6 for logarithmic wind profile, and substitute in the altitudes corresponding to t_1 and t_2 which are labeled z_1 and z_2 respectively.

$$L_{\mathrm{app}} = \frac{1}{V_{\mathrm{v}}^*} \int_{z_1}^{z_2} V_{\mathrm{h}}^* + W(z) \, dz \tag{52}$$

At this point, an additional feature of the wind model must be incorporated. The value of the wind magnitude $W(z)$ is assumed not to change sign with changing altitude; *i.e.,* the output values of the function $W(z)$ should be either all positive or all negative over the range of altitude from z_{start} to the surface. The complication is that at values of altitude z close to zero, the sign of the natural logarithm function does change sign. Therefore, for this wind model, the value of the wind magnitude $W(z)$ shall be considered to be zero for values of altitude z close to zero. It can be verified that for $z = -e^{-\beta/\alpha}$, the value of wind magnitude $W(z) = \alpha \ln(-z) + \beta = 0$; therefore, $W(z) = 0$ for $|z| < |e^{-\beta/\alpha}|$. The integral in Eq. (52) can then be separated into two components: one with a non-zero value of $W(z)$ for $-e^{-\beta/\alpha} > z > z_1$ and one with a zero value for $W(z)$ for $0 > z > -e^{-\beta/\alpha}$. Note that z_2 is the touchdown point, so that its altitude is zero.

$$L_{\text{app}} = \frac{1}{V_{\text{v}}^*}\left\{\int_{z_1}^{-e^{-\beta/\alpha}} V_{\text{h}}^* + \alpha \ln(-z) + \beta \; dz\right\} + \frac{1}{V_{\text{v}}^*}\int_{-e^{-\beta/\alpha}}^{0} V_{\text{h}}^* \; dz \tag{53}$$

For the both integrals, perform variable substitution as in Eq. (31) on page 14 to account for the fact that altitude is represented as a negative number in the sign convention in use.

$$L_{\text{app}} = \frac{-1}{V_{\text{v}}^*}\left\{\int_{-z_1}^{e^{-\beta/\alpha}} V_{\text{h}}^* + \alpha \ln z' + \beta \; dz'\right\} - \frac{1}{V_{\text{v}}^*}\int_{e^{-\beta/\alpha}}^{0} V_{\text{h}}^* \; dz' \tag{54}$$

Evaluate the definite integrals and the limits of integration and group terms.

$$L_{\text{app}} = \frac{V_{\text{h}}^*}{V_{\text{v}}^*}\left(-e^{-\beta/\alpha} - z_1 + e^{-\beta/\alpha}\right) - \frac{\beta}{V_{\text{v}}^*}\left(e^{-\beta/\alpha} + z_1\right) - \frac{\alpha}{V_{\text{v}}^*}\left[z' \ln z' - z'\right]_{-z_1}^{e^{-\beta/\alpha}} \tag{55}$$

$$L_{\text{app}} = \left(\frac{-V_{\text{h}}^* - \beta}{V_{\text{v}}^*}\right) z_1 - \frac{\beta e^{-\beta/\alpha}}{V_{\text{v}}^*} - \frac{\alpha}{V_{\text{v}}^*}\left[e^{-\beta/\alpha} \ln e^{-\beta/\alpha} - e^{-\beta/\alpha} + z_1 \ln(-z_1) - z_1\right] \tag{56}$$

$$L_{\text{app}} = \left(\frac{-V_{\text{h}}^* + \alpha - \beta}{V_{\text{v}}^*}\right) z_1 + \frac{\alpha e^{-\beta/\alpha}}{V_{\text{v}}^*} - \frac{\alpha}{V_{\text{v}}^*}\left[z_1 \ln(-z_1)\right] \tag{57}$$

Using the assumption of constant descent rate V_{v}^*, the altitude z_1 at the beginning of the approach segment is just the time duration of the approach segment multiplied by the vertical velocity, expressed as a negative number:

$$z_1 = -T_{\text{app}} V_{\text{v}}^* \tag{58}$$

Substituting Eq. (58) into Eq. (57), a final expression is obtained in terms of values α, β, V_{v}^*, and V_{h}^* that are known *a priori*, and value T_{app} that is easily computed using the known value T_{turn} and an estimated value for T_{downwind}.

$$L_{\text{app}} = (V_{\text{h}}^* - \alpha + \beta) T_{\text{app}} + \frac{\alpha e^{-\beta/\alpha}}{V_{\text{v}}^*} + \alpha T_{\text{app}} \ln(T_{\text{app}} V_{\text{v}}^*) \tag{59}$$