Nine-degree of Freedom Modeling and Flight Dynamic Analysis of Parafoil Aerial Delivery System

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Abstract

Because of its good aerodynamic performance and excellent controllability, the PADS which using the parachute like a wing has been widely used. To reach the accurate delivery target, the flight dynamic performance of PADS is very important. It is the foundation of the aerial delivery system’s controllability and stability analysis. In this paper, we analyse the PADS’ flight dynamic performance by simplified the system at 9-DOF and numerical modeling based on it. The system flight dynamic performance at the station of one side edge deflection and both side edge deflection have been compared. The results indicate that this paper’s modeling and dynamic analysis way is viable.

Keywords: PADS; flight dynamic performance; 9-DOF; deflection

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>F</td>
<td>force</td>
</tr>
<tr>
<td>I</td>
<td>moment of inertia</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
</tr>
<tr>
<td>M</td>
<td>moment</td>
</tr>
<tr>
<td>p, q, r</td>
<td>roll, pitch, and yaw rates</td>
</tr>
<tr>
<td>T</td>
<td>conversion matrix</td>
</tr>
</tbody>
</table>

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1. Introduction

Nowadays, accurate delivery of a payload by parachute is a requirement in both the space and military fields. Because of its good aerodynamic performance and excellent controllability, the PADS which using the parachute like a wing has been widely used[1]. To reach the accurate delivery target, the flight dynamic performance of PADS is very important. It is the foundation of the aerial delivery system’s controllability and stability analysis. Fig.1.(a) is a photo of PADS.

Fig.1. (a)PADS; (b) simplified PADS.
2. Numerical modeling

We analyse the aerial delivery system’s flight dynamic performance by simplified the system at 9-DOF[2,3] and numerical modelling based on it. The 9-DOF is as that three of freedom each for rotational motion of the parafoil, and payload, and three DOF for translational motion of the confluence point of the lines. That means both the Parafoil and the payload are free to rotate about joint C, but are constrained by the internal joint force at C. It can describe the two-body problem well. Fig. 1. (b) shows the simplified PADS, and also the force acting on it. And the 9-DOF needs to assume the PADS as:

- The Parafoil is spread wise symmetric, and the canopy shape is unchangeable when inflation full
- The moment of inertia change at the center of mass is ignored when doing the edge deflection control
- Each of the Parafoil and payload is rigid body

Now, we can get the 9-DOF dynamic equations from Fig.1. (b) by using the Newton’s law[4]. Because of the paper’s length, the dynamic equations are just given as follows without detailed deduction.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} u_x \\ v_x \\ w_x \end{bmatrix}$$ (1)

$$\begin{bmatrix} \phi_x \\ \psi_x \end{bmatrix} = \begin{bmatrix} 1 & \sin \varphi_x \tan \theta \phi \cos \varphi_x \tan \theta \phi \\ 0 & \cos \varphi_x \end{bmatrix} \begin{bmatrix} \phi_p \\ \psi_p \end{bmatrix}$$ (2)

$$\begin{bmatrix} mT_{x\theta} \\ mT_{y\theta} \end{bmatrix} = \begin{bmatrix} -m \ddot{r}_x \\ -m \ddot{r}_y \end{bmatrix} \times \begin{bmatrix} \ddot{r}_x \\ \ddot{r}_y \end{bmatrix}$$ (4)

And:

$$B_1 = F_{a} + F_{g} - m \ddot{r}_x \ddot{r}_x$$; \hspace{1cm} B_2 = F_{a} + F_{g} - m \ddot{r}_y \ddot{r}_y$$; \hspace{1cm} B_3 = M_{x} + M_{y} = \ddot{r}_x \ddot{r}_x$$; \hspace{1cm} B_4 = M_{x} + M_{y} = \ddot{r}_y \ddot{r}_y$$

These 21 equations totally contain 21 variables. 18 variables of system situation: the location of confluence point C in Eq. (1), the payload’s posture in Eq. (2), the Parafoil’s posture in Eq. (3), and the velocity of C angle velocity of payload and Parafoil in Eq. (4). And 3 variables of the constraint force at point C. So these equations make up a first order ordinary differential equations, could be solved by Runge-Kutta way.

3. Parameters

3.1. Geometry parameters

The dynamic equations above could be normally used for general hinged system of two rigid bodies. When used for PADS, we should build up the relationship between the equations ‘parameters and PADS’ parameters. A group of PADS’ geometry parameters is shown in fig.2. (a). The Parafoil is simplified as a curve surface, as the line blacked. The mass is assumed as concentrated in the surface, and the Parafoil coordinate is also built along the direction of the curve, by right hand rule as shown in fig.2. (a).

That is the bending height, the vertical distance from the curve’s top to bottom point. And b is the canopy span, c is the canopy chord, t is the thickness of canopy, \(l_{front}\) is the line length at the front of the canopy symmetric surface, \(l_{back}\) is the line length at the back of the canopy symmetric surface. These parameters could construct the shape of the canopy when inflation full. The payload location is relating with the length of connection tapes. These geometry
parameters, with the mass and the moment of inertia parameters, describe the whole PADS system. Then we should use these parameters to calculate the constant variables $m_b$, $I_b$, $r_{cb}$, $m_p$, $I_p$, $r_{cp}$ in the equations.

The location of gravity center of Parafoil is assumed at 40% of the chord along $x$ axis direction, 50% of the bending height along $z$ axis direction. Then as shown in Fig.2. (b)(side elevation of the Parafoil symmetric surface), the location of gravity center could be solved out by chord $c$, $l_{front}$ and $l_{back}$.

![Fig.2. (a)geometry parameters; (b)the location of gravity center.](image)

So the vector $r_{pc}$ could be represented in the Parafoil coordinate as:

$$x_{pc} = 0.4c - t, \ y_{pc} = 0, \ z_{pc} = \sqrt{l_{front}^2 - t^2 - 0.5a}; \text{ where } t = \frac{l_{front}^2 - l_{back}^2 + c^2}{2c}.$$

The payload gravity center could be given by the real location. Then using the connection tape’s length, the relatively location from payload mass center $b$ to linked point $C$ could be calculated. If the payload is cubic, the length, wide and height each is $l_x$, $l_y$, $l_z$, the mass is equal distributed, and the length of connection tape is $l_b$.

Then the vector $r_{pc}$ could be represented in the payload coordinate as:

$$x_{pc} = 0, \ y_{pc} = 0, \ z_{pc} = -0.5l_z - \sqrt{l_y^2 + l_z^2}.  \frac{l_z^2 - l_y^2}{4}.$$

### 3.2. Mass parameters

The mass of parafoil and payload could be measured. If there is no measured number, the parafoil and lines could be assumed as a cubic to simplify the calculation. The cubic length is $c$, wide is $b$ and height is $a+t$. Then the moment of inertia according to the gravity center is:

$$I_x = \frac{m_p}{12}((a+t)^2 + b^2), \ I_y = \frac{m_p}{12}((a+t)^2 + c^2), \ I_z = \frac{m_p}{12}(b^2 + c^2)$$
Similarly, the moment of inertia of payload according to the gravity center is:

\[ I_x = \frac{m_p}{12} (l_y^2 + l_z^2), I_y = \frac{m_p}{12} (l_x^2 + l_z^2), I_z = \frac{m_p}{12} (l_x^2 + l_y^2) \]

3.3. Aerodynamic force

The aerodynamic force in reference [2] is used as to compare the results in section 4.

3.4. Apparent mass

The apparent mass is the same as Lissaman and Brown’s way [5].

4. Results

The dynamic analysis process of the PADS is: the Parafoil is assumed inflation full at the start of calculation, but the PADS is not gliding steadily. When the PADS become steady, the control [6, 7] will be added in. The location, velocity and posture of the Parafoil and payload are memorized. The PADS ‘parameters are assumed in Table 1.

<table>
<thead>
<tr>
<th>parameter</th>
<th>( c )</th>
<th>( t )</th>
<th>( a )</th>
<th>( b )</th>
<th>( L_{\text{front}} )</th>
<th>( L_{\text{back}} )</th>
<th>( l_a )</th>
<th>( l_b )</th>
<th>( l_c )</th>
<th>( l_d )</th>
<th>( m_p )</th>
<th>( m_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>3.8m</td>
<td>0.68m</td>
<td>1.1m</td>
<td>7.7m</td>
<td>7.554m</td>
<td>8.041m</td>
<td>1.0m</td>
<td>0.6m</td>
<td>0.5m</td>
<td>0.4m</td>
<td>5.0kg</td>
<td>135kg</td>
</tr>
</tbody>
</table>

We compare the results with reference [2]. The first control input is both edge deflection 100% for 25s, after 50s from beginning. The calculated results (marked by cal. in fig.) and reference results (marked by ref. in fig.) are compared in fig. 3. It could be seen in fig. 3 that these results are almost the same, payload pitch angle, Parafoil pitch angle, horizontal velocity and vertical velocity.
Then we compare the results at one side edge deflection control. The control input is also after 50s from beginning, right side, but it is holding until the calculation is finished. Fig. 4 shows the one side deflection results compared with calculated and reference. These results, payload pitch angle, roll angle, parafoil pitch angle, roll angle and the horizontal, vertical velocity of the confluence point C, are also nearly the same.

So through the compare of calculated results and reference results, at both side deflection and one side deflection, there are all nearly the same, don’t have much difference. Therefore, these results indicate that this paper’s modeling and dynamic analysis way used to simulation the PADS’ is viable.
Fig. 4. One side deflection results compared

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References